Profondità di moto uniforme.

Legge di Chezy: $U = C\sqrt{g R i_f}$

Sostituendo in:

$$Q = U\Omega = \Omega C\sqrt{g R i_f} \Rightarrow \frac{Q}{\sqrt{g i_f}} = \Omega C\sqrt{R}$$

Abbiamo $\Omega, R \propto Y$, cioè:

$$\frac{Q}{\sqrt{g i_f}} = \left[C\,\Omega\sqrt{R}\right]_{Y=Y_u}$$

La profondità di moto uniforme Y_u soddisfa l'uguaglianza.

Si cerca lo zero di:

$$F(Y) = \frac{Q}{\sqrt{g i_f}} - \left[C\,\Omega\sqrt{R}\right]_{Y=Y_u}$$

Nel caso di canale rettangolare infinitamente largo $\left(\frac{Y}{b} \ll 1\right)$:

$$C\,\Omega\sqrt{R} = \frac{k_s R^{1/6}}{\sqrt{g}} \cdot bY\sqrt{\frac{b\cdot Y}{b+2Y}} = \frac{k_s}{\sqrt{g}}\, bY \cdot \left(\frac{bY}{b+2Y}\right)^{2/3} =$$

$$= \frac{k_s}{\sqrt{g}} \cdot bY \cdot \left(\frac{Y}{1+\underbrace{2\frac{Y}{b}}_{\simeq 0}}\right)^{2/3} = \frac{k_s}{\sqrt{g}} \cdot b\, Y^{5/3}$$

Quindi:

$$\frac{Q}{\sqrt{\cancel{g}\, i_f}} = \frac{k_s}{\cancel{\sqrt{g}}}\, b \cdot Y^{5/3} \Rightarrow Q = k_s\sqrt{i_f}\, b\, Y^{5/3}$$

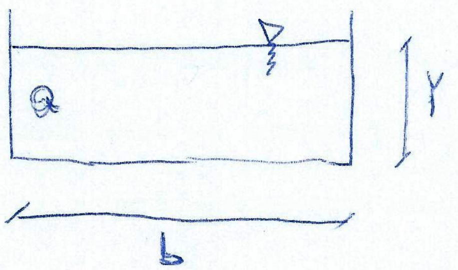
$$\Rightarrow Y_u = \left(\frac{Q}{k_s\sqrt{i_f}\, b}\right)^{3/5}$$

Inoltre:

$$Q = C\,\Omega\sqrt{g R i_f} = \frac{k_s}{\cancel{\sqrt{g}}} R^{1/6} \cdot \Omega\sqrt{\cancel{g} R i_f} = k_s\sqrt{i_f}\,\Omega R^{2/3} =$$

$$= k_s\sqrt{i_f}\,\Omega \cdot \left(\frac{\Omega}{B}\right)^{2/3} = k_s\sqrt{i_f}\,\Omega^{5/3} B^{-2/3}$$

Sezione rettangolare:



$$Q = Q\,(b, Y, K_s, i_f)$$

$$Q = K_s \sqrt{i_f}\ \Omega^{5/3} \cdot B^{-2/3}$$

$$\Omega = bY \qquad B =$$

$$Q = K_s \sqrt{i_f}\,(bY)^{5/3} \cdot (b+2Y)^{-2/3} =$$

$$= K_s \sqrt{i_f}\ \frac{(bY)^{5/3}}{(bY)^{2/3}} \cdot \frac{(bY)^{2/3}}{(b+2Y)^{2/3}} = K_s \sqrt{i_f} \cdot bY\ R^{2/3}$$

Per sezioni infinitamente larghe $(b \gg Y)$:

$$R = \frac{bY}{b+\underbrace{2Y}_{=0}} \simeq Y \Rightarrow Q = K_s \sqrt{i_f}\, b \cdot Y \cdot Y^{2/3} = K_s \sqrt{i_f}\, b\, Y^{5/3}$$

$$\Rightarrow Y_u = \left(\frac{Q}{b K_s \sqrt{i_f}} \right)^{3/5}$$

Y_u risulta sottovalutata poiché abbiamo tolto l'altezza Y delle pareti.

Processo iterativo:

$$F(Y) = \frac{Q}{K_s \sqrt{i_f}} - (bY)^{5/3} \cdot (b+2Y)^{-2/3} = \frac{Q}{K_s \sqrt{i_f}} - \Omega^{5/3} \cdot (B)^{-2/3}$$

$$F'(Y) = -\frac{5}{3}\,\Omega^{5/3-1} \cdot B^{-2/3} \cdot \frac{d\Omega}{dY} + \frac{2}{3}\, B^{-2/3-1}\, \frac{dB}{dY} \cdot \Omega^{5/3} =$$

$$= -\frac{5}{3}\,\Omega^{5/3-1} \cdot B^{-2/3} \cdot b + \frac{2}{3}\, B^{-5/3}\, 2 \cdot \Omega^{5/3} =$$

$$= \Omega^{5/3} \cdot B^{-2/3} \left[-\frac{5}{3}\, \frac{\cancel{b}}{\cancel{b}Y} + \frac{4}{3}\, \frac{1}{2Y+b} \right] =$$

$$= (bY)^{5/3} (b+2Y)^{-2/3} \left[-\frac{5}{3Y} + \frac{4}{3(2Y+b)} \right]$$

Si usano:

$$Y_{n+1} = Y_n - \frac{F(Y_n)}{F'(Y_n)} \qquad \varepsilon = \left| \frac{Y_{n+1} - Y_n}{Y_n} \right| < 10^{-3}$$

Esercizio moto uniforme sezione rettangolare.

Progetto:

$k_s = 35\ m^{1/3}\ s^{-1}$

$i_f = 0{,}0035$

$b = 70\ m$

I Alveo rettangolare infinitamente largo: $Y_u = \left(\frac{Q}{b\, k_s \sqrt{i_f}}\right)^{3/5}$

Al variare di Q:

$Q\ [m^3/s]$	100	200	300	400	500	800	1000	1200
$Y_u\ [m]$	a) 0,800	1,213	1,547	1,839	2,102	2,784	3,186	3,555

II Caso reale:

a) $Q = 100\ m^3/s$ $\quad Q = k_s \cdot \sqrt{i_f} \cdot (bY)^{5/3} \cdot (b+2Y)^{-2/3}$

$$F(Y) = \frac{Q}{k_s\sqrt{i_f}} - (bY)^{5/3} \cdot (b+2Y)^{-2/3}$$

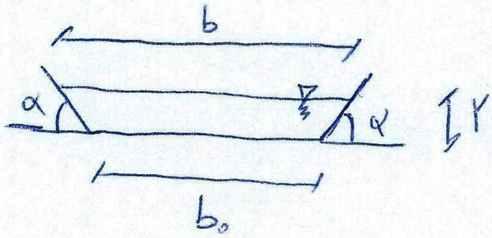
$$F'(Y) = -\frac{5}{3}(bY)^{5/3-1} \cdot b \cdot (b+2Y)^{-2/3} + \frac{2}{3}(b+2Y)^{-2/3-1} \cdot 2 \cdot (bY)^{5/3} =$$

$$= (bY)^{5/3} \cdot (b+2Y)^{-2/3} \left[\frac{4}{3(b+2Y)} - \frac{5}{3Y}\right]$$

Tabella:

n	$Y_n [m]$	$B [m]$	$\Omega [m^2]$	$F(Y) [m]$	$F'(Y)$	ε
0	0,800	71,6	56	0,757	-98,152	-
1	0,808	71,616	56,56	-0,031	-98,78	$2{,}23 \cdot 10^{-4}$
2	0,808	71,616	—	—	—	~ 0

Alveo trapezio:



$$F(Y) = \frac{Q}{k_s \sqrt{i_f}} - \Omega^{5/3} \cdot B^{-2/3}$$

$$\Omega = b_0 Y + 2 \frac{Y}{2} \cdot \frac{Y}{\sin\alpha} \cdot \cos\alpha = b_0 Y + \frac{Y^2}{2\,\mathrm{tg}\alpha} 2 = b_0 Y + \frac{Y^2}{\mathrm{tg}\alpha}$$

$$b = b_0 + 2 \frac{Y}{\sin\alpha} \cos\alpha = b_0 + \frac{2Y}{\mathrm{tg}\alpha} \qquad B = b_0 + \frac{2Y}{\sin\alpha}$$

$$\Rightarrow F(Y) = \frac{Q}{k_s \sqrt{i_f}} - \left(b_0 Y + \frac{Y^2}{\mathrm{tg}\alpha}\right)^{5/3} \cdot \left(b_0 + \frac{2Y}{\sin\alpha}\right)^{-2/3}$$

$$\Rightarrow F'(Y) = -\frac{5}{3}\left(b_0 Y + \frac{Y^2}{\mathrm{tg}\alpha}\right)^{5/3-1} \cdot \left(b_0 + \frac{2Y}{\sin\alpha}\right)^{-2/3} \cdot \left(b_0 + \frac{2Y}{\mathrm{tg}\alpha}\right) +$$

$$+ \frac{2}{3}\left(b_0 + \frac{2Y}{\sin\alpha}\right)^{-2/3-1}\left(b_0 Y + \frac{Y^2}{\mathrm{tg}\alpha}\right)^{5/3}\left(b_0 + \frac{2Y}{\mathrm{tg}\alpha}\right) =$$

$$= \left(b_0 Y + \frac{Y^2}{\mathrm{tg}\alpha}\right)^{5/3} \cdot \left(b_0 + \frac{2Y}{\sin\alpha}\right)^{-2/3}\left[-\frac{5}{3} \cdot \frac{b_0 + \frac{2Y}{\mathrm{tg}\alpha}}{\left(b_0 Y + \frac{Y^2}{\mathrm{tg}\alpha}\right)} + \frac{4}{3\left(b_0 + \frac{2Y}{\sin\alpha}\right)\sin\alpha}\right]$$

Consideriamo:

$b_0 = 5\,m \quad Q = 20\,m^3/s \quad k_s = 30\ m^{1/3} s^{-1} \quad \alpha = 45° \quad i_f = 0{,}0030$

Tabelle:

n	Y_n[m]	b[m]	A[m²]	B[m]	F(Y)	F'(Y)	ε
0	1,705	8,411	11,432	9,822	-0,477	-13,081	--
1	1,669	8,338	11,131	9,721	-0,011	-12,847	0,021
2	1,668	8,336	11,122	9,718	0,002	-12,840	0,0006

Quindi $Y = 1{,}668\ m$. Verifichiamo:

$$Q = k_s \cdot \sqrt{i_f} \cdot \left(b_0 Y + \frac{Y^2}{\mathrm{tg}\alpha}\right)^{5/3} \cdot \left(b_0 + \frac{2Y}{\sin\alpha}\right)^{-2/3} =$$

$$= 30\ m^{1/3}s^{-1} \cdot \sqrt{0{,}003}\left(5 \cdot 1{,}668\,m^2 + \frac{1{,}668^2\,m^2}{\mathrm{tg}45°}\right)^{5/3} \cdot \left(5\,m + \frac{2 \cdot 1{,}668\,m}{\sin 45°}\right)^{-2/3} =$$

$$= 30\ m^{1/3}s^{-1} \cdot \sqrt{0{,}003} \cdot 55{,}418\ m^{10/3} \cdot 0{,}220\ m^{-2/3} =$$

$$\simeq 20{,}00\ m^3/s$$

Realisticamente prendiamo $Y = 1{,}67\ m$